Structural Constraints on Ontological Unification: A Formal Model of Multiple Domains

Abstract

The assumption that reality is ontologically homogeneous—that is, reducible to a single fundamental structure—underlies much of contemporary metaphysics, including variants of physicalism, monism, and structural realism. However, persistent difficulties concerning cross-domain identity, structural preservation across levels, and ontological dependence suggest intrinsic limits to unifying approaches.

This paper develops a formal model in which reality is represented as a system of multiple ontological domains that are locally coherent and globally irreducible, characterized by partial transitions and invariant-preserving relations.

From a set of minimal structural conditions—ontological heterogeneity, compositional loss, local invariance, and metastructural dependence—it is shown that invariant preservation is not stable under composition, and that no global system can guarantee structural coherence across domains.

It follows that no ontological reduction function can preserve invariants globally, implying the impossibility of complete structural preservation, the irreducibility of domains, and the absence of a universal criterion of identity.

The contribution of the paper is structural rather than empirical: it identifies formal constraints that any ontological model must satisfy, showing that global ontological unification is incompatible with systems characterized by heterogeneity and compositional loss.

1 Introduction

The assumption that reality admits a unified ontological description constitutes one of the most persistent commitments in contemporary metaphysics. In different philosophical traditions, this commitment takes distinct but structurally equivalent forms: physicalism holds that all phenomena are reducible to the physical domain; priority monism maintains that reality is fundamentally a single entity or structure; ontological structural realism proposes that the global structure of reality suffices to describe all phenomena.

Despite their differences, these approaches share a central premise: the existence of a global ontological structure capable of preserving all relevant properties of phenomena. In formal terms, this premise implies the existence of a reduction function F such that, for any ontological domains Di and Dj, F performs a structural translation preserving relevant invariants.

More precisely, the ontological reduction hypothesis requires the existence of a global function F satisfying the following conditions:

Totality: for every state s in Di, F(s) is defined in Dj
Preservation: if a property I is an invariant in Di, then I is preserved under F in Dj
Compositionality: for any domains Di, Dj, and Dk, the composition of translations preserves invariants, that is, Fik = Fjk composed with Fij

These conditions can be expressed as follows:

For every s in Si, there exists F(s) in Sj
For every invariant I, if I(s) holds, then I(F(s)) holds
For every sequence of domains Di, Dj, and Dk, Fik(s) = Fjk(Fij(s))

These conditions express the requirement that the ontological structure of reality be globally consistent, independently of the path of translation between domains.

However, this expectation faces systematic difficulties that are not merely empirical or contingent, but structural.

First, transitions between ontological domains often involve loss of information. Formally, there exist invariants I and transitions Tij such that I does not belong to Inv(Tij), implying that properties defined in Di are not preservable in Dj.

Second, the notion of cross-domain identity is problematic. If identity were global, there would exist a relation Ψ that is reflexive, symmetric, and transitive across all domains. However, since Ψ depends on local invariants, this transitivity fails along chains of transitions.

Third, relations of ontological dependence cannot be internalized within a single system. The existence of a foundational structure requires an operator Φ that does not belong to any domain Di.

These difficulties motivate a central question: is there a global function capable of fully preserving the ontological structure of reality? If not, any homogeneous ontological model is, by definition, insufficient.

This paper adopts the opposite hypothesis: reality is composed of structurally distinct ontological domains whose interconnections do not fully preserve their properties.

The central argument is developed through the following sequence:

Ontological domains are structurally heterogeneous
Transitions between domains are partial and non-invertible
The composition of transitions entails structural loss
Structural loss implies incompatibility of invariants
Incompatibility prevents the construction of a global system of invariants

To formalize this argument, it is shown that if compositional loss occurs, then there exists an invariant I such that I belongs to Inv(Tij) but not to Inv(Tjk composed with Tij). It follows that global preservation is impossible.

The aim of this paper is not to propose a specific ontology, but to identify structural constraints that any ontological model must satisfy. In particular, it is shown that the impossibility of global preservation does not depend on empirical limitations, but on formal properties of the composition of heterogeneous structures.

This paper adopts a regime of structural formalization in which concepts are defined by their relations and operational properties, without presupposing a complete axiomatization in first-order logic. The goal is not to construct a closed logical system, but to identify structural constraints that emerge from the interaction between domains, transitions, and invariants.

This paper presents the first formal component of the Foundations of Constitutive Ontology research program. Its purpose is to establish structural constraints on ontological unification. The broader formal architecture underlying this program will be developed in future foundational work.

2 Formal Structure of the Model

The ontological system is defined as a structure:

$R = (D, T, \delta, A, \Phi, \Psi, Inv)$

where each component plays a specific role in representing the structure of reality.

Formally, R can be understood as a partially structured system with categorical features, in which objects correspond to ontological domains and morphisms correspond to partial transitions.

The set $D = \{Di\}$ denotes a family of ontological domains.

Each domain Di is represented as a structure:

$Di = (Si, Li, Ci, Pi, Vi)$

where:

Si is a non-empty set
Li is closed under logical implication
Ci is a subset of Si
Vi is a valuation function from Pi × Si to truth values

Proposition 1 — Local structural closure

For every domain Di, if s belongs to Si and φ belongs to Li, then:

If s satisfies φ and φ implies ψ, then s satisfies ψ.

Proof

This follows directly from the closure of Li under logical implication.

Proposition 2 — Structural irreducibility

There exist domains Di and Dj such that there is no bijection f from Si to Sj that preserves the valuation function Vi.

Proof (constructive)

Consider two domains with fundamentally different structural bases. In one domain, predicates are defined over quantitative properties; in another, predicates are defined over semantic relations.

Any bijection between the underlying sets would fail to preserve the valuation structure, since quantitative predicates cannot, in general, be mapped to semantic predicates in a way that preserves their evaluation across all elements.

Therefore, no bijection can preserve the valuation function across such domains.

3 Ontological Transitions

Formally, a transition Tij is a partial function:

Tij: Dom(Tij) ⊆ Si → Sj

where Dom(Tij) denotes the subset of states in Si for which the transition is defined.

3.1 Preservation of invariants

An invariant I is preserved by Tij if, for every state s in Dom(Tij), the following holds:

if I(s) holds, then I(Tij(s)) also holds.

This definition captures the notion of local structural preservation associated with a transition.

3.2 Theorem — Inevitable Structural Loss

In any system exhibiting ontological heterogeneity, there exist transitions Tij and Tjk such that:

Inv(Tjk composed with Tij) is strictly contained in Inv(Tij).

Proof

Consider three ontological domains Di, Dj, and Dk with distinct structural bases.

By the assumption of ontological heterogeneity, there exist invariants defined in Di that do not admit full structural correspondence in Dj. That is, there exists at least one invariant I such that:

I belongs to Inv(Tij), but its interpretation in Dj depends on additional structure or undergoes modification.

Now consider the composition Tjk composed with Tij.

Even if I is preserved under Tij, its preservation under Tjk depends on its stability within Dj. Since the structure of Dj does not guarantee the preservation of all invariants originating in Di, there exist invariants whose preservation fails at the second stage.

Formally, there exists a state s in Dom(Tij) such that:

I(s) holds
I(Tij(s)) holds
but I(Tjk(Tij(s))) does not hold

Hence, I does not belong to Inv(Tjk composed with Tij).

Moreover, this situation is not exceptional. It follows from the absence of complete structural correspondence between domains, which is a direct consequence of ontological heterogeneity.

Therefore, the set of invariants preserved by the composition is strictly smaller than the set preserved by the first transition.

3.3 Structural Interpretation of Loss

Structural loss does not arise from contingent limitations, but from the incompatibility between ontological structures.

Transitions between domains are not mere re-descriptions, but transformations that alter the conditions under which invariants are defined.

The composition of transitions amplifies this effect, as each step may introduce new constraints or eliminate previously preserved properties.

3.4 Structured Examples

Example 1 — Physical to Biological

Physical domain:
Invariant: conservation of energy

Biological domain:
Invariant: adaptive functionality

In the transition from the physical to the biological domain, conservation of energy may be partially preserved. However, adaptive functionality cannot be reduced to local physical properties without structural loss.

Under further composition, the preservation of these properties becomes increasingly restricted.

Example 2 — Syntactic to Semantic

Syntactic domain:
Invariant: formal structure

Semantic domain:
Invariant: meaning

There is no function that preserves both formal structure and meaning under arbitrary composition, since meaning depends on relations that are not internal to the syntactic system.

Thus, syntactic invariants do not guarantee semantic invariants under composed transitions.

3.5 Composition as Structural Filtering

The composition of transitions can be understood as a filtering operation on invariants.

Formally:

Inv(Tjk composed with Tij) is contained in Inv(Tij) intersect Inv(Tjk)

and, in general, this inclusion is strict.

This implies that composition not only preserves shared invariants, but eliminates those that are not compatible across both stages.

3.6 Structural Consequence

There is no sequence of transitions capable of preserving all invariants of an initial domain.

For any chain of transitions:

T1, T2, ..., Tn

the set of preserved invariants decreases along the chain.

This establishes a fundamental structural limitation: global preservation of invariants is impossible in ontologically heterogeneous systems.

4 Metric Structure and Accessibility

The function δ introduces a notion of distance between ontological domains. This distance should not be interpreted as a metric in the classical sense, but as a measure of structural cost associated with transitions between domains.

Formally, δ is defined as a function from D × D to the non-negative real numbers, satisfying, for all Di, Dj, and Dk in D:

Identity:
δ(Di, Di) = 0

Non-negativity:
δ(Di, Dj) ≥ 0

Oriented subadditivity:
if there exist admissible composable transitions Tij and Tjk, then:
δ(Di, Dk) ≤ δ(Di, Dj) + δ(Dj, Dk)

Symmetry is not required. In general:
δ(Di, Dj) is not equal to δ(Dj, Di)

This asymmetry is structurally significant. It captures situations in which the transition from Di to Dj involves a loss of information that cannot be recovered in the reverse direction. Accordingly, δ measures not only distance, but also structural irreversibility.

The function δ can also be interpreted as a minimal structural cost: for each pair of domains, it measures the least amount of transformation required to render states of Di admissible in Dj, subject to structural constraints, laws, and conditions of existence.

Proposition 4.1 — Asymmetry implies irreversibility

If δ(Di, Dj) is not equal to δ(Dj, Di), then there exists at least one invariant I such that:

I belongs to Inv(Tij) and does not belong to Inv(Tji)

Proof

If the distances were symmetric, the structural cost of transformation would be equivalent in both directions, allowing for potential reversibility. Asymmetry implies that the inverse transformation entails additional loss. Therefore, there exists at least one invariant that cannot be recovered in the reverse direction.

4.1 Interpretation of ontological distance

The function δ can be understood as the minimal cost required to transform a structure in Di into an admissible structure in Dj. This cost is not merely quantitative, but reflects changes in structural configuration, logical validity, and conditions of existence.

Low values of δ(Di, Dj) indicate structural proximity, that is, the existence of transitions that preserve a relatively large set of invariants. High values indicate the need for substantial transformation, typically involving significant loss of invariants.

The asymmetry of δ implies that the cost of translation from Di to Dj may differ substantially from the inverse direction. This formalizes the idea that some domains are structurally richer or more restrictive than others.

Proposition 4.2 — Relationship between cost and structural loss

For any positive ε, there exists a threshold such that, if δ(Di, Dj) exceeds ε, then there exists a domain Dk closer to Di such that:

the cardinality of Inv(Tij) is strictly smaller than that of Inv(Tik)

Interpretation: the greater the ontological distance, the smaller the set of preservable invariants.

4.2 Relationship between distance and invariants

There is an inverse relationship between δ and the set of invariants preserved by a transition. For admissible transitions Tij:

as δ(Di, Dj) increases, the size of Inv(Tij) tends to decrease.

This relationship is not strictly monotonic, but reflects a fundamental structural tendency: greater ontological distance corresponds to reduced preservability of structural properties.

However, this relationship cannot be adequately captured by cardinality alone. A more refined notion of preservation is required.

4.3 Degree of preservation and the role of τ

The function δ is an abstract quasi-metric on the set of domains, without assuming a full topological structure. The function τ is defined as a partial measure on sets of invariants, induced by a monotonic measure function μ.

The function μ assigns values between 0 and 1 to sets of invariants and is monotonic with respect to structural inclusion.

For each admissible transition Tij, the degree of preservation is defined as:

τij(Tij) equals μ of Pres(Tij)

and the maximal degree of preservation is given by the supremum of these values over all admissible transitions.

The introduction of τ is structurally necessary. Identifying the set of preserved invariants is not sufficient to characterize preservation under composition.

This is because:

the composition of transitions produces non-uniform loss of invariants
different invariants have different structural relevance
compositional degradation cannot be captured by simple counting

Therefore, any adequate account of structural preservation requires a measure on invariants.

In particular, the analysis of compositional effects along chains of transitions depends on τ.

Proposition 4.3 — Preservation bound under distance

For any Di and Dj, there exists a decreasing function g such that:

τij is bounded above by g of δ(Di, Dj)

Interpretation: ontological distance imposes an upper bound on structural preservation.

4.4 Structured example

Consider three domains:

D1: physical
D2: biological
D3: mental

In general:

δ(D1, D2) is less than δ(D1, D3)

Therefore:

τ12 is greater than τ13

This captures the intuition that the transition from the physical to the mental domain involves greater structural loss than the transition from the physical to the biological domain.

4.5 Accessibility function

The accessibility function A, defined from D × D to the interval [0, 1], represents the relative feasibility of transitions between domains.

It is defined as:

A(Di, Dj) equals f of δ(Di, Dj) and τij

where:

f decreases with respect to δ
f increases with respect to τ

Proposition 4.4 — Monotonicity of accessibility

If:

δ(Di, Dj) is greater than δ(Di, Dk)
and τij is less than τik

then:

A(Di, Dj) is less than A(Di, Dk)

4.6 Structural role of the quasi-metric

The introduction of δ prevents trivialization of the system. Without a notion of structural cost, it would be possible to assume that any domain can be translated into any other without significant loss.

The function δ enforces three structural features:

non-uniformity of ontological structure
irreversibility of transformations
impossibility of global equivalence between domains

The interaction between δ and τ establishes a fundamental constraint: structural cost limits the degree of preservation achievable under transitions.

This shows that the ontological system is not only qualitatively heterogeneous, but also quantitatively constrained by its internal transformation structure.

Accordingly, reality cannot be modeled as a homogeneous space, but must be understood as a stratified and asymmetric structure.

5 Invariants and Identity

Invariants constitute the conceptual core of the model. While transitions represent structural transformation, invariants capture what remains stable under such transformations.

Formally, an invariant I is a property defined on ontological states such that, for a valid transition Tij, the following holds:

for every s in the domain of Tij, if I(s) holds, then I(Tij(s)) also holds.

Theorem 5.1 — Contextual Dependence of Invariants

There is no universal invariant independent of transitions.

Proof

Suppose that there exists an invariant I such that, for every transition Tij, I belongs to Inv(Tij). This would imply that I is preserved under all transitions, and therefore across any composition of transitions.

However, by the principle of compositional loss established earlier, there exist invariants that fail to be preserved under composition. Therefore, no invariant can be preserved universally across all transitions.

This yields a contradiction. Hence, no universal invariant exists.

5.1 Nature of invariants

Invariants are not global properties. Their definition depends simultaneously on:

the structure of the source domain
the structure of the target domain
the nature of the transition under consideration

Therefore, invariants are inherently contextual.

Formally:

Inv(Tij) is not equal to Inv(Tjk) in general.

5.2 Typology of invariants

In addition to the distinctions introduced earlier, invariants can be classified according to their structural dependence:

Structural invariants: depend on σ
Logical invariants: depend on λ
Existential invariants: depend on κ

Accordingly, an invariant can be expressed as:

I(s) equals a function of σ, λ, and κ

5.3 Structure of invariants

A partial order relation is defined on the set of invariants such that:

I is less than or equal to J if, for every transition, the preservation of J implies the preservation of I.

5.4 Preservational realization

An invariant I is realizable if and only if there exists a transition Tij such that:

for every state s, if I(s) holds, then I(Tij(s)) also holds.

5.5 Invariants and identity

Partial identity Ψ is defined as follows:

Ψ(s, t) holds if and only if there exists an invariant I such that I(s) and I(t) both hold, and I belongs to Inv(Tij).

Proposition 5.2 — Non-transitivity of identity

If Ψ(s, t) and Ψ(t, u), it does not follow that Ψ(s, u).

Proof

Each identity relation is grounded in a set of invariants that may differ across transitions. There is no guarantee that the invariants supporting Ψ(s, t) and Ψ(t, u) overlap. Therefore, transitivity cannot be ensured.

5.6 Structural incompatibility

Two invariants I and J are structurally incompatible if and only if there exists no transition Tik such that both I and J belong to Inv(Tik).

Theorem 5.3 — Existence of incompatible invariants

There exist invariants I and J such that I is incompatible with J.

Proof

By compositional loss and the absence of global invariant preservation, there exist invariants that cannot be jointly preserved under any admissible transition. Hence, structurally incompatible invariants necessarily exist.

6 Ontological Foundation

The operator Φ represents a meta-structural condition necessary for coherence among ontological domains.

Formally, Φ does not belong to the set of domains D; that is, Φ is not an element of D and cannot be reduced to any internal structure defined by a specific domain.

Φ functions as a structural admissibility operator, determining the conditions under which transitions between domains are possible. In particular, the existence of an admissible transition Tij presupposes that the constraints imposed by Φ are satisfied.

Thus, Φ does not describe states, properties, or local laws, but establishes coherence conditions that make relations between domains possible.

6.1 Meta-structural nature of Φ

Unlike the internal elements of the system, Φ cannot be characterized as a structural object at the same level as the domains. It operates on the system as a whole, regulating compatibility between heterogeneous structures.

This implies that Φ is not accessible through any internal transition, nor can it be reconstructed from the composition of such transitions.

In structural terms, Φ performs three fundamental functions:

it establishes admissibility conditions for transitions between domains
it ensures minimal consistency between irreducible structures
it prevents trivialization of the system through structural collapse

Without Φ, there is no criterion for determining when a transition is structurally valid, and the ontological system becomes indeterminate.

6.2 Theorem — Irreducibility of Φ

There is no domain Di such that Φ can be defined entirely within Li.

Proof

Suppose, for contradiction, that there exists a domain Di in which Φ can be fully defined in terms of its internal structure Li.

This would imply that all coherence conditions between domains could be expressed within Di. In particular, for any domains Dj and Dk, the admissibility of transitions Tjk would be determined by properties internal to Di.

Hence, Di would function as a universal domain, capable of internalizing all structural relations in the system.

However, by Theorem 8.2 (non-globalizability of invariants), there is no global system capable of preserving compositional coherence across domains.

Therefore, no domain can internalize all necessary structural conditions.

This yields a contradiction. Hence, Φ cannot be fully defined within any domain.

6.3 Consequence — Structural exteriority

From the irreducibility of Φ it follows that:

Φ is structurally exterior to the system of domains.

This exteriority is not spatial or causal, but formal: Φ does not belong to the structure it regulates, but constitutes a condition for its possibility.

Accordingly, no internal description of the system can be complete, since it necessarily depends on conditions that cannot be expressed within any specific domain.

6.4 Φ as a condition of possibility

Φ acts as a condition of possibility for transitions between domains.

More precisely, for any admissible transition Tij, its existence depends on the satisfaction of constraints imposed by Φ.

Without Φ:

there is no admissibility criterion
there is no coherence between domains
there is no guarantee of structural consistency

Thus, Φ not only regulates the system, but grounds the very possibility of relations between domains.

6.5 Relationship between Φ and non-globalizability

The impossibility of constructing a global system of invariants (Theorem 8.2) implies that coherence cannot be guaranteed internally.

Φ therefore emerges as the only structure capable of fulfilling this role without violating ontological heterogeneity.

In other words:

non-globalizability prevents internal unification
Φ guarantees structural coherence at a meta-level

This shows that Φ is not an arbitrary addition to the model, but a necessary structural consequence.

6.6 Structural interpretation

Φ can be understood as the level at which the conditions for coherence between heterogeneous structures are established.

It does not describe the content of domains, but regulates the conditions under which such content can be related.

This implies that:

Φ is not observable within any domain
Φ is not derivable from invariants
Φ cannot be eliminated without compromising the coherence of the system

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6.7 Synthesis

The operator Φ plays a central role in the model:

it is not reducible to any domain
it is not internalizable
it is not optional

It constitutes the minimal structural condition required for the existence of an ontological system composed of distinct domains.

Without Φ, the system collapses into incoherence or triviality.

7 Structural Conditions

The conditions presented in this section should not be understood as arbitrary assumptions, but as necessary consequences of the structure defined by domains, partial transitions, and invariant preservation.

Each condition expresses a structural constraint arising from the interaction between ontological heterogeneity and compositional dynamics.

7.1 Ontological heterogeneity

Ontological heterogeneity is defined as the absence of a function between domains that preserves all structural invariants simultaneously. Formally, two domains Di and Dj are heterogeneous if there is no function f from Si to Sj such that every invariant defined on Si is preserved under f.

Theorem 7.1 — Non-existence of global isomorphism

There is no function f from Si to Sj that preserves all invariants.

Proof

Suppose, for contradiction, that such a function f exists between two domains Si and Sj.

Then, for every invariant I defined on Si, if I(s) holds, then I(f(s)) also holds. This would require the structure of Sj to be sufficiently rich to represent all invariants of Si without loss.

Since the argument is general, this would imply that any domain could be translated into any other without structural loss.

However, by the Structural Loss Theorem (Section 3), there exist invariants that fail to be preserved under composition of transitions.

Therefore, no function can preserve all invariants between distinct domains.

7.2 Compositional loss

Theorem 7.2 — Structural non-transitivity

The preservation of invariants is not transitive.

Proof

Consider three domains Di, Dj, and Dk, and two admissible transitions Tij and Tjk.

Suppose that an invariant I is preserved under Tij; that is, if I(s) holds, then I(Tij(s)) also holds. Suppose further that I is preserved under Tjk.

For preservation to be transitive, I would also need to be preserved under the composition Tjk composed with Tij.

However, by Theorem 3.2, there exist invariants that fail to be preserved under composition. Accordingly, there exists a state s such that:

I(s) holds
I(Tij(s)) holds
but I(Tjk(Tij(s))) does not hold

Therefore, preservation is not transitive.

7.3 Local invariance

Proposition 7.3

There does not exist a global set of invariants Inv_global such that:

for every transition Tij, Inv_global is contained in Inv(Tij)

Proof

Suppose that such a set exists.

Then every invariant in Inv_global would be preserved under any transition and, therefore, under any composition of transitions.

However, structural loss under composition implies that invariants may fail to be preserved along chains of transitions.

Therefore, no set of invariants can be preserved globally.

7.4 Metastructural dependence

Proposition 7.4

Φ is a necessary condition for the existence of admissible transitions.

Proof

An admissible transition must satisfy structural coherence conditions between domains.

Since domains are structurally distinct and no global system of invariants exists, these conditions cannot be defined internally within a domain.

Therefore, the admissibility of transitions depends on conditions external to the system of domains.

These conditions are precisely those represented by Φ.

7.5 Interdependence of conditions

Theorem 7.5 — Structural impossibility of unification

The combination of the preceding conditions implies that no consistent global ontological structure exists.

Proof

From ontological heterogeneity, it follows that there is no global isomorphism between domains.

From compositional loss, it follows that preservation of invariants is not transitive.

From local invariance, it follows that no global set of invariants can be preserved.

From metastructural dependence, it follows that coherence cannot be guaranteed internally.

If a consistent global ontological structure existed, it would have to:

preserve invariants
guarantee compositional coherence
be independent of external conditions

However, each of these requirements is ruled out by the conditions above.

Therefore, no such structure can exist.

7.6 Methodological observation

The structural conditions presented are independent in the sense that none of them, taken in isolation, implies the impossibility of ontological unification.

In particular, ontological heterogeneity does not by itself exclude the existence of partially preserving translations, and compositional loss does not prevent local preservation of invariants.

The impossibility of unification arises only from the combination of these conditions under composition, and not from any one of them taken in isolation.

This ensures that the result is not analytic with respect to the premises, but emerges from the interaction of distinct structural constraints.

8 Key Results

This section presents the logical core of the paper. The aim is to demonstrate, based on the previously established structural conditions, that it is not possible to construct a global system of invariants capable of preserving compositional coherence across ontological domains.

To make the argument explicit, we first introduce the notion of strong compositional preservation:

A set of invariants Inv* is compositionally preservable if, for any admissible transitions Tij and Tjk, the following holds: for every invariant I in Inv*, if I is preserved by Tij and by Tjk, then I is also preserved by the composition Tjk composed with Tij.

The nonexistence of such a set is established below.

8.1 Lemma — Loss implies incompatibility

If structural loss occurs under composition of transitions, then there exist incompatible invariants.

Proof

Consider two admissible transitions Tij and Tjk and an invariant I preserved by Tij. Thus, for every state s in the domain of Tij, if I(s) holds, then I(Tij(s)) also holds.

Suppose that I is not preserved under the composition Tjk composed with Tij. Then there exists a state s such that I(s) holds, but I(Tjk(Tij(s))) does not hold.

This shows that I is not stable under composition.

Now consider an invariant J preserved by Tjk. Since I fails under composition while J is defined relative to the second transition, there is no guarantee that both can be jointly preserved under a single composite transition.

In particular, any invariant K intended to capture both I and J would fail to be preserved, since the failure of I under composition entails the failure of any stronger invariant implying it.

Therefore, I and J are incompatible.

8.2 Theorem 1 — Non-globalizability of invariants

There exists no global system of invariants capable of preserving compositional consistency.

Proof

Suppose, for contradiction, that there exists a global system of invariants Inv_global such that, for every admissible transition Tij, Inv_global is contained in Inv(Tij).

Then all invariants in Inv_global would be preserved under any admissible transition and, therefore, under any composition of transitions.

In particular, for any Tij and Tjk and any invariant I in Inv_global, we would have:

I belongs to Inv(Tij)
 I belongs to Inv(Tjk)
 therefore, I must belong to Inv(Tjk composed with Tij)

However, by Lemma 8.1, there exist incompatible invariants that cannot be preserved simultaneously under composition.

Furthermore, from the definition of the degree of preservation τ, τij represents the maximal degree of invariants preservable between Di and Dj. Under composition, the degree of preservation satisfies:

τik is less than or equal to the minimum of τij and τjk

Thus, preservation does not increase under composition.

In addition, from the structural relationship between ontological cost δ and preservation τ established in Section 4, an increase in ontological distance corresponds to a decrease in the degree of preservation. Since chains of transitions imply accumulation of structural cost, compositions lead to further reduction in preservation.

Therefore, there exist chains of transitions along which the degree of preservation strictly decreases, implying the existence of invariants that fail to be preserved.

Hence, there exists at least one invariant I in Inv_global such that:

I belongs to Inv(Tij)
 I does not belong to Inv(Tjk composed with Tij)

This contradicts the assumption that Inv_global is preserved under all transitions.

Therefore, no such global system exists.

8.3 Corollary 1 — Impossibility of global preservation

There is no global function F capable of preserving all invariants across domains.

If such a function existed, it would induce a global set of preserved invariants defined by:

Inv_F consists of all invariants I such that, for every state s, if I(s) holds, then I(F(s)) also holds.

This set would necessarily be preserved under composition, which contradicts Theorem 8.2.

8.4 Corollary 2 — Ontological irreducibility

There is no universal domain that internalizes all ontological domains.

Suppose such a domain D* exists. Then, for every domain Di, there would be a transition from Di to D* preserving relevant invariants.

This would imply that all invariants could be expressed within D*, which is equivalent to the existence of a global system of invariants.

Since such a system does not exist, no such domain can exist.

8.5 Corollary 3 — Non-existence of global identity

There is no universal identity relation that holds across all domains.

Formally, there is no relation Ψ such that, for all states s, t, and u across domains, the following conditions hold simultaneously:

Reflexivity: Ψ(s, s)
Symmetry: Ψ(s, t) implies Ψ(t, s)
Transitivity: if Ψ(s, t) and Ψ(t, u), then Ψ(s, u)

The failure of transitivity follows from the incompatibility of invariants and the effects of compositional degradation.

8.6 Interpretation of the results

These results show that the impossibility of ontological unification does not arise from empirical limitations, but from structural constraints.

More precisely, the definition of partial transitions, together with compositional loss, the bounded nature of preservation through τ, and the increase in structural cost δ along chains of transitions, jointly imply that any attempt to preserve global structure leads to contradiction.

Thus, any unified ontology would require abandoning at least one of these structural conditions, resulting either in trivialization or inconsistency.

8.7 Structural example

Consider two domains:

A physical domain, in which a relevant invariant is conservation of energy
A mental domain, in which a relevant invariant is semantic intentionality

Both are stable within their respective domains. However, when attempting to establish a transition between them, no invariant can simultaneously preserve both properties.

Consider the transition chain:

Physical → Biological → Mental

Conservation of energy may be partially preserved in the transition from physical to biological domains, but it is not preserved in the same structural sense in the transition from biological to mental domains.

Conversely, intentionality cannot be reconstructed from physical structure without loss.

Formally, this is reflected in:

a progressive increase in ontological distance δ
a corresponding decrease in the degree of preservation τ

This illustrates concretely how invariants fail to remain stable under composition, and why no global system of invariants can be constructed.

9 Categorical Reconstruction

The ontological structure defined above can be reinterpreted in categorical terms.

A categorical structure Ont is defined as follows:

Objects: ontological domains Di
Morphisms: admissible transitions Tij

The composition of morphisms is partial and defined when applicable.

More precisely, the composition Tjk composed with Tij is defined only when the codomain of Tij coincides with the domain of Tjk, and when structural admissibility conditions are satisfied.

This restriction implies that Ont is not, in general, a category in the full classical sense, but a partial categorical structure in which composition is not universally defined.

9.1 Properties of the categorical structure

The structure Ont has the following fundamental characteristics:

partial composition: not all pairs of morphisms are composable
absence of general inverses: there is generally no inverse morphism for every transition
structural degradation under composition: composition reduces the set of preserved invariants

If Tij and Tjk are composable, then:

the set of invariants preserved by the composition is contained in the intersection of the invariants preserved by each transition

Moreover, by structural loss, this inclusion is generally strict.

This implies that composition is not structurally neutral, but introduces cumulative loss.

9.2 Absence of a terminal object

A terminal object would be a domain D* such that, for every domain Di, there exists a unique morphism from Di to D*.

Suppose such an object exists.

Then, for any domains Di and Dj, compositions of morphisms through D* would be well-defined and structurally compatible.

This would imply that the relevant invariants of all domains are preserved under transitions to D*.

In particular, there would exist a set of invariants common to all domains and preserved by all such transitions.

However, structural loss under composition implies that invariants fail to be preserved along chains of transitions.

Therefore, no global set of invariants can exist.

Hence, the existence of a terminal object leads to contradiction.

It follows that Ont has no terminal object.

9.3 Absence of global colimits

Colimits allow the construction of an object that unifies a diagram of objects and morphisms.

For a colimit to exist, it must be possible to reconcile all transitions in the diagram into a single object while preserving their essential structural properties.

Consider a diagram involving multiple domains and transitions.

For a colimit to exist, there must be a domain Dc such that all transitions factor through Dc in a consistent manner.

However, structural loss implies that invariants preserved in different parts of the diagram are not necessarily compatible under composition.

Thus, there is no set of properties that can be preserved simultaneously across all transitions.

Therefore, no structure can unify all domains without loss in a way compatible with the definition of a colimit.

It follows that Ont does not admit global colimits.

9.4 Non-existence of a global structure-preserving functor

Consider a functor F from Ont to some category C.

For F to preserve the structure of the system, it must map each morphism Tij to a morphism F(Tij) that preserves the relevant invariants.

Moreover, this preservation must be compatible with composition, meaning that images of compositions correspond to compositions of images.

However, structural loss implies that invariant preservation is not stable under composition.

Therefore, no mapping can preserve invariants consistently across all transitions.

It follows that no global structure-preserving functor exists.

9.5 Categorical consequence

From the preceding results, it follows that the structure Ont does not admit any form of global unification within the categorical framework.

More precisely:

there is no terminal object
there are no global colimits
there are no global structure-preserving functors

These limitations are not contingent, but follow directly from the structural properties of the system, in particular from invariant incompatibility and compositional degradation.

Thus, the structure of reality cannot be represented as a single object within a larger categorical framework without loss of structural information.

9.6 Structural interpretation

The categorical interpretation reinforces the central thesis of the paper: reality does not admit global structural compression.

The absence of terminal objects prevents reduction of all domains to a single structure.
The absence of colimits prevents construction of a unified whole from local structures.
The absence of structure-preserving functors prevents complete structural translation between levels.

Taken together, these results show that ontological heterogeneity is not merely descriptive, but a necessary formal consequence.